

Warm-Up

Determine the Domain, Vertical Asymptotes, and Holes of the following:

$$f(x) = \frac{3x^2 - 21x + 36}{x^2 - 5x + 6}$$

Domain: $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

V.A : at $x=2$

Hole $(3, -3)$

$$\frac{3(x-4)(x-3)}{(x-2)(x-3)} = \frac{3(x-4)}{x-2}$$
$$\frac{3(3-4)}{3-2}$$

$$\#6 \quad \frac{(2x+1)(x+5)}{(2x+1)(x-3)} = \boxed{\frac{x+5}{x-3}}$$

Simplified

Domain:

$$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)$$

V.A at $x=3$

$$\frac{-\frac{1}{2} + 5}{-\frac{1}{2} - 3}$$

Hole: $(-\frac{1}{2}, -\frac{9}{7})$

$$\frac{\frac{9}{2}}{-\frac{7}{2}} \cdot \frac{2}{-\frac{9}{7}}$$

$$\frac{9}{7}$$

Objectives

I can...

- Determine Horizontal Asymptotes of Rational Functions
- Determine Slant Asymptotes of Rational Functions

V.A

Holes

Horizontal Asymptotes

Compare the degree of Numerator vs Denominator

3 Cases

Degree

1) Top = Bottom

H.A. ($y =$)

Fraction of Leading Coefficients

2) Bottom > Top

$y = 0$

3) Top > Bottom

No H.A. (Could be Slant!)

Examples

1.) $f(x) = \frac{x^2 - 16}{-3x^2 + 6x + 9}$ H.A at $y = -\frac{1}{3}$

2) $f(x) = \frac{x+4}{-3x^2 - 3x + 18}$ H.A at $y = 0$

3) $f(x) = \frac{x^3 - 2x^2 - 8x}{4x^2 + 12x + 8}$ No H.A.

Examples

$$1) \ f(x) = -\frac{3}{x^{\cancel{0}} + 2} \quad \text{H.A. at } y=0$$

$$2) \ f(x) = \frac{x^{\cancel{3}} - 16x}{4x^{\cancel{2}} + 12x} \quad \text{No H.A.}$$

$$3) \ f(x) = \frac{3x^{\cancel{3}} - 9x^2 + 6x}{ix^{\cancel{3}} - 4x^2 + 3x} \quad \text{H.A. at } y=3$$

Slant Asymptotes

- Occur when the Numerator is one degree higher than the Denominator

To Find:

- Use Long Division (or Synthetic)
- Ignore the Remainder
- S.A. is $y = \text{Quotient}$

Ex. 1 $f(x) = \frac{2x^2 - 5x + 5}{x - 2}$

2]
$$\begin{array}{r} 2 \quad -5 \quad 5 \\ \underline{-} \quad 4 \quad -2 \\ 2 \quad -1 \quad \cancel{5} \end{array}$$

$2x - 1$

S.A at $y = 2x - 1$

Ex. 2 $f(x) = \frac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$ S.A at
 $y = 2x + 7$

$$\begin{array}{r} 2x+7 \\ \hline x^2 - 3x + 2 \end{array} \overline{)2x^3 + x^2 - 8x - 4} \\ - (2x^3 - 6x^2 + 4x) \\ \hline 7x^2 - 12x - 4 \\ \hline 7x^2 - 21 + 14 \\ \hline \end{array}$$

Ex.3 $f(x) = \frac{x^3 + x^2 - 4}{x^2 + 1}$

S.A at
 $y = x + 1$

$$\begin{array}{r} x^2 + 0x + 1 \quad | \quad x^3 + x^2 + 0x - 4 \\ \underline{- (x^3 + 0x^2 + x)} \\ \underline{x^2 - x - 4} \\ \underline{- x^2 - 0x + 1} \end{array}$$

Ex. 4

$$f(x) = \frac{x^2 - 6x + 7}{x + 5}$$

$$\begin{array}{r} -5 \\ \underline{-5} \quad | \quad -6 \quad 7 \\ \quad \quad -5 \quad 55 \\ \hline \quad \quad \quad -11 \end{array}$$

S.A at
 $y = x - 11$

Ex. 5 $f(x) = \frac{5x^3 - 8}{x^2 + 3x - 1}$

Ex. 6 $f(x) = \frac{3x^5 + 2x^2}{x^2 - 7x + 5}$

